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Magnetic susceptibility of a semiconductor superlattice under parallel electric and magnetic fields

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Abstract. We have calculated the magnetic susceptibility χ of the degenerate electron gas in a semiconductor superlattice placed in parallel quantizing electric (E) and magnetic (H) fields directed along the superlattice growth axis. In this case the electrons have a purely discrete energy spectrum consisting of Landau and Wannier–Stark levels. This leads to: (i) the dependence of the monotonic part of the diamagnetic susceptibility of the electron gas on the electric field; (ii) the rise of a new oscillation period of χ as a function of H different from that of the de Haas–van Alphen oscillations; and (iii) the oscillatory behaviour of χ as a function of E with a periodicity in $\sqrt{E}$. It is shown that in weak magnetic fields the Wannier–Stark quantization results in a step-like dependence of the paramagnetic susceptibility of the electron gas on the electric field strength.

1. Introduction

In the past few years, there has been a steadily increasing interest in the electronic properties of semiconductor superlattices (SL) subjected to a strong electric field. This is accounted for by the fact that the SL is a unique object in studying a fundamental physical phenomenon—the quantization of electron motion in a periodic potential under an external electric field. This phenomenon was predicted by Wannier [1] in the early 1960s and since then has been a topic of considerable interest and controversy. But it has not been until very recently that essential progress has been made in understanding this problem owing to advances in microfabrication technology of semiconductor SL having artificial periodicity. The Wannier–Stark quantization in these structures has been intensively studied both theoretically [2–9] and experimentally [9–18]. The results obtained till now (for a review, see [19, 20]) not only prove convincingly the existence of electric-field-induced localization of electron eigenstates and, connected with it, the Wannier–Stark ladder spectrum in the semiconductor SL, they also show good potential for various applications in optoelectronics (see, for example, [21] and references therein).

With the application of a strong magnetic field perpendicular to the SL layers and a strong electric field directed along the SL axis, the in-plane motion of the electrons is quantized into Landau levels, whereas the electron motion parallel to the growth axis is quantized into Wannier–Stark levels. This completely discrete energy spectrum leads to highly unusual optical properties of the SL, which have been studied in a number of theoretical and experimental works [22–24].

One can expect that various thermodynamic properties of the electron gas in the SL will change essentially when electric and magnetic fields cause quantization of electron motion. In this connection it is of great interest to investigate the behaviour of the magnetic

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susceptibility of the degenerate electron gas, particularly the oscillations of magnetic susceptibility under magnetic field variation (the de Haas–van Alphen effect). The latter has long been widely used as a powerful tool for studying the electron energy spectrum of bulk semiconductors and metals [25]. So far there have been several theoretical [26–30] and experimental [31–33] studies of this effect in low-dimensional electron systems such as the inversion layer on the Si surface and the semiconductor SL GaAs–AlGaAs. In the latter the de Haas–van Alphen oscillations have been observed by making use of a SQUID (superconducting quantum interference device) magnetometer with a sensitivity of 10^{-10} J T^{-1}, sufficient to measure the magnetization in SL consisting of 172 layers with total area of 33 cm^2.

In this paper, we present a theory of the magnetic susceptibility χ of the degenerate electron gas in a semiconductor SL in the presence of quantizing electric (E) and magnetic (H) fields parallel to the SL axis. As will be shown, a purely discrete electron energy spectrum consisting of Landau and Wannier–Stark levels leads to the dependence of the monotonic part of the diamagnetic susceptibility of the electron gas on the electric field and to the rise of a new oscillation period of χ as a function of H depending on the electric field strength and differing from that of the de Haas–van Alphen oscillations. It is also shown that, on varying the electric field, oscillations of χ with a periodicity in $\sqrt{E}$ will be observed, their period being proportional to the magnitude H. In weak magnetic fields, as we will see further, the existence of the Wannier–Stark ladder levels manifests itself in a step-like dependence of the paramagnetic susceptibility of the electron gas on the electric field. This dependence arises from the peculiarities of the density of states of the electrons in the SL in the presence of a quantizing electric field.

2. The energy spectrum and the eigenstates of the electrons in the SL under parallel electric and magnetic fields

Let us consider an n-type semiconductor SL in which the Fermi level ζ_0 is in the conduction band in the absence of external fields. We assume the SL period d to be considerably greater than the lattice constant a of the host crystal. This permits the envelope-function method [19] to be used for describing the SL electronic states. Within the framework of this approach the action of the potential with natural period a is taken into account by means of an effective-mass approximation, whereas the artificial potential of the SL is regarded as acting only upon the envelope function, which is slowly varying at the scale of a.

We take the SL growth axis as the z axis and choose the vector potential A of the uniform magnetic field in the Landau gauge $A_x = A_z = 0$, $A_y = Hx$. The dynamics of an electron interacting with electric and magnetic fields parallel to the z axis is described by the Hamiltonian

$$\mathcal{H} = \frac{p_x^2}{2m_\perp} + \frac{(p_y + m_\perp \omega_H x)^2}{2m_\perp} + \frac{p_z^2}{2m_\parallel} + V(z) + eEz \tag{1}$$

where p_x, p_y and p_z are operators of the electron momentum component, $m_\perp$ and $m_\parallel$ are the components of the effective mass of an electron perpendicular and parallel to the SL axis, respectively, $\omega_H = eH/m_\perp c$ is the cyclotron frequency and $V(z) = V(z+d)$ is the SL potential.

It is easy to see that the variables in the Schrödinger equation with the Hamiltonian (1) can be separated. Indeed, the motion along the y axis is free, in the x direction we get an equation for the quantum harmonic oscillator with the equilibrium centre at the point $x_0 = -a_H^2 k_y$ ($a_H = \sqrt{c\hbar/eH}$ is the magnetic length), whereas in the z direction we get an equation describing the motion of an electron with effective mass $m_\parallel$ in the electric field E and the SL potential $V(z)$. The latter equation can be easily solved in a one-band model of tightly bound electrons [34, 35]. We make use of the results obtained in these papers supposing the electrons occupy (up to the Fermi level) only the ground (lower) miniband with a width Δ. We also assume that the following conditions are satisfied:

$$\omega_H \tau \gg 1 \qquad \omega_E \tau \gg 1 \qquad \hbar\omega_H \gg T \qquad \Delta_g \gg \hbar\omega_E \gg T \tag{2}$$

where $\omega_E = eEd/\hbar$ is the Wannier–Stark frequency, τ is the electron relaxation time, T is the temperature in energy units and Δ_g is the width of the first minigap. The latter of the conditions (2) permits Zener tunnelling and thermal excitations of electrons from ground to higher minibands to be ignored. In this case the normalized eigenstates and the corresponding eigenvalues of the Hamiltonian (1) can be presented in the form

$$\psi_\alpha(r) = \frac{1}{\sqrt{L_y}} e^{ik_y y}\Phi_n\left(\frac{x - x_0}{a_H}\right)\sum_l J_{l-m}(\Delta/2\hbar\omega_E)\varphi_{\mathrm{loc}}(z - ld) \tag{3}$$

$$\varepsilon_\alpha = \varepsilon_{nm} = (n + \tfrac{1}{2})\hbar\omega_H + m\hbar\omega_E. \tag{4}$$

Here $\alpha \equiv (k_y, n, m)$ denotes the set of quantum numbers determining the state of the electron (n is the number of Landau levels, m enumerates the Wannier–Stark ladder levels), L_y is the normalization length in the y direction, $\Phi_n(x)$ is the oscillator wavefunction, $J_l(z)$ is a Bessel function of the first kind and of integer index l and $\varphi_{\mathrm{loc}}(z - ld)$ is the isolated SL quantum well eigenstate determined by the explicit form of the $V(z)$ potential.

As is clearly seen from expression (3), the envelope function, while oscillating, is rapidly decreasing with the increase of the electric field strength, so that an electron in the mth Wannier–Stark ladder level turns out to be spatially localized around $z = md$ with extension of the order $\Delta/\hbar\omega_E$. Further, we confine ourselves to regarding a most realistic case when the parameter $\Delta/\hbar\omega_E$ is not too large and, hence, only a comparatively small number of Wannier–Stark levels can be arranged on the miniband width.

Note that for an infinite SL the quantum number $m = 0, \pm1, \pm2, \ldots$ and, therefore, levels with negative energy appear in the electron spectrum (4). Physically it is accounted for by the fact that electrons can move to infinity in the negative z direction and, hence, find themselves in a region of arbitrarily large negative potential energy. The real SL is bounded, of course, and in order to simulate the boundary at $z = 0$, preserving expression (4) for the energy spectrum at the same time, we can make use, following [35], of an approach taken from other situations in which infinitely negative-energy fermion states appear [36, 37]. Namely, we postulate the existence of a filled-up infinite reservoir ('Dirac sea') of negative energy levels. Then, without loss of generality, we can regard the quantum number m in expression (4) as taking only positive integer values. An SL having another boundary (in the positive z direction) will further be taken into account indirectly in evaluating the electron density of states by multiplying it by the factor L_z/d equal to the number of SL quantum wells. It holds provided that $eEL_z \gg \Delta/2$, which minimizes edge effects [22, 38]. The energy level system of the SL electrons in this case is schematically depicted in figure 1.

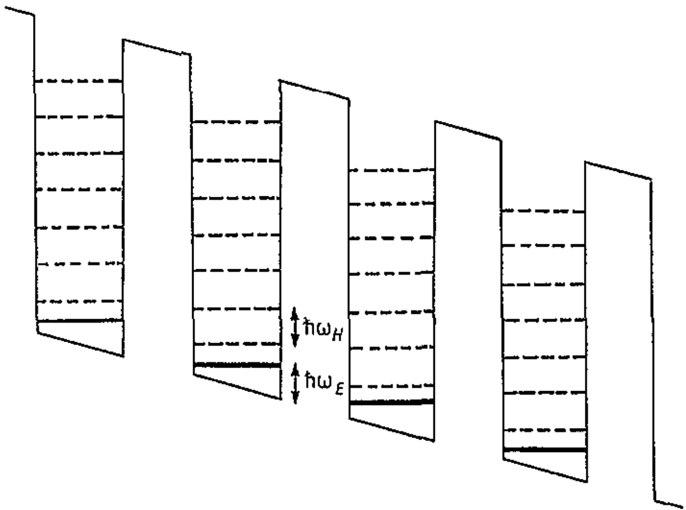

Figure 1. Schematic conduction-band edge profiles for the SL under quantizing electric and magnetic fields applied parallel to the SL axis. The full and broken lines in the quantum wells represent the Wannier–Stark and Landau levels, respectively.

3. The thermodynamic potential

We consider now the thermodynamic potential Ω, the derivatives of which with respect to the magnetic field (at fixed values of the temperature T and of the chemical potential ζ of electrons) determine the magnetization and magnetic susceptibility of the electron gas. The expression for Ω has the form

$$\Omega = -2T \sum_{k_y} \sum_{n} \sum_{m} \ln \left[1 + \exp\left(\frac{\zeta - \varepsilon_{nm}}{T} \right) \right] \tag{5}$$

where the magnitude $\zeta = \mu + eEz$ (μ is the chemical potential proper) and is determined from the condition that the local electron density

$$n_e(\zeta) = V_0^{-1} \int D(\varepsilon) \left[1 + \exp\left(\frac{\varepsilon - \zeta}{T} \right) \right]^{-1} \mathrm{d}\varepsilon \tag{6}$$

does not change on applying the electric and magnetic fields, e.g. from the condition

$$n_e(\zeta) = n_e^{(0)}(\zeta_0) \tag{7}$$

where $n_e^{(0)}(\zeta_0)$ is the electron density in the SL at $E = H = 0$. In formula (6), V_0 is the volume of the system and $D(\varepsilon)$ is the electron density of states, which, in the case of spectrum (4), has the form

$$D(\varepsilon) = (V_0/\pi d a_H^2) \sum_{n} \sum_{m} \delta(\varepsilon - \varepsilon_{nm}). \tag{8}$$

In an SL with one type of charge carrier, when the impurities are entirely ionized, condition (7) is obviously satisfied practically in the whole of the SL specimen, except for the narrow interface regions, the contribution of which to Ω is insignificant.

The evaluation of the thermodynamic potential with the help of formula (5) can be carried out using two methods: (i) by the standard method going back to the classical Landau paper (see [25]) and based on the Poisson summation formula for singling out the oscillating parts from thermodynamic quantities; or (ii) by the method suggested by Sondheimer and Wilson [39] (see also [40]) based on the theorem connecting density of states with the Laplace transformation of the statistical sum. In the case of the purely discrete spectrum of electrons that we consider here, the first of the above-mentioned methods leads to the representation of Ω in the form of a double Fourier series, which is very complicated for analysis in the general case. Therefore, we make use of the second method, devoid of this deficiency. This method has one more advantage, which allows one to express the thermodynamic potential for the degenerate electron gas by means of the classical statistical sum $Z(\lambda)$, which, in the case of spectrum (4), is easy to calculate:

$$Z(\lambda) = (V_0/2\pi d a_H^2)[1-\exp(-\lambda\hbar\omega_H)]^{-1}[\exp(\lambda\hbar\omega_E)-1]^{-1}\exp(-\lambda\hbar\omega_H/2) \tag{9}$$

where $\lambda = T^{-1}$. Following the approach elaborated in [39, 40] we represent Ω in the form

$$\Omega = 2\int_0^{\infty} z(\varepsilon_\alpha)[\partial f_0(\varepsilon_\alpha)/\partial\varepsilon_\alpha]\,\mathrm{d}\varepsilon_\alpha \tag{10}$$

where $f_0(\varepsilon_\alpha)$ is the Fermi–Dirac distribution function and the function $z(\varepsilon_\alpha)$ is expressed by the Mellin integral

$$z(\varepsilon_\alpha) = (2\pi\mathrm{i})^{-1}\int_{\gamma-\mathrm{i}\infty}^{\gamma+\mathrm{i}\infty} Z(\lambda)\lambda^{-2}\exp(\lambda\varepsilon_\alpha)\,\mathrm{d}\lambda. \tag{11}$$

Here the integration is carried out in the complex λ plane along the straight line parallel to the imaginary axis and passing on the right of it (the constant $\gamma > 0$).

Taking into account (9), it is easy to see that the integrand in (11) has singularities at the points $\lambda = 0$, $\lambda = \lambda_s = \mathrm{i}2\pi s/\hbar\omega_H$ ($s = \pm1, \pm2, \ldots$) and $\lambda = \lambda_l = \mathrm{i}2\pi l/\hbar\omega_E$ ($l = \pm1, \pm2, \ldots$). The point $\lambda = 0$ is a pole of fourth order, while the character of the other singular points depends, generally speaking, on the arithmetic properties of the number ω_H/ω_E; namely, if the frequencies ω_H and ω_E are incommensurate (i.e. ω_H/ω_E is an irrational number), the integrand in (11) has poles of first order at the points λ_s and λ_l, but if the mentioned frequencies are commensurate (i.e. ω_H/ω_E is rational), the integrand in (11) has poles of second order at the points λ_s or λ_l (which are one and the same). Meanwhile, it is quite clear that the dependence of such a macroscopic quantity as the magnetization of the electron gas on the strength of the fields must be continuous and, since any irrational number is well approximated by a rational fraction, we can confine ourselves, without loss of generality, to regarding the commensurate case when ω_H/ω_E is rational. There is an additional argument in favour of this statement: namely, if Ω is evaluated in a different way, based on the Landau method, the case of irrational ω_H/ω_E is in no way singled out.

The further evaluation of Ω by means of formula (10) is carried out directly. Closing the integration contour in (11) by an infinite semicircle in the left half-plane ($\mathrm{Re}\,\lambda < \gamma$), we calculate the integral by making use of Cauchy's residue theorem. Substituting the result obtained in (10) and carrying out the integration over $\mathrm{d}\varepsilon_\alpha$, we get the following expressions for the monotonic and oscillating parts of Ω:

$$\Omega^{\mathrm{mon}} = -\frac{m_\perp V_0}{6\pi\hbar^3\omega_E d}(\zeta-\hbar\omega_E/2)[\zeta(\zeta-\hbar\omega_E)+(\pi T)^2-(\hbar\omega_H/2)^2] \tag{12}$$

$$\Omega^{\rm osc} = \frac{m_\perp V_0 \omega_H^3}{2\pi^4 \omega_E d} \sum_l \left\{ (-1)^l l^{-3} \Psi_1\left(\frac{2\pi^2 l T}{\hbar\omega_H}\right) \left[\left(\frac{\pi l}{\hbar\omega_H}\right)\left(\zeta - \frac{\hbar\omega_E}{2}\right)\cos\left(\frac{2\pi l\zeta}{\hbar\omega_H}\right) \right.\right.$$
$$\left.\left. - \Psi_2\left(\frac{2\pi^2 l T}{\hbar\omega_H}\right)\sin\left(\frac{2\pi l \zeta}{\hbar\omega_H}\right)\right]\right\} \tag{13}$$

where

$$\Psi_1(x) = x/\sinh x \simeq \begin{cases} 1 - x^2/6 & x \ll 1 \\ 2x\exp(-x) & x \gg 1 \end{cases} \tag{14}$$

$$\Psi_2(x) = \tfrac{1}{2}(1 + x\coth x) = \begin{cases} 1 + x^2/6 & x \ll 1 \\ x/2 & x \gg 1. \end{cases} \tag{15}$$

The expressions (12) and (13) contain an unknown quantity ζ. To exclude it we make use of the equality

$$n_e(\zeta) = -V_0^{-1}(\partial\Omega/\partial\zeta)_{T,H,E} \tag{16}$$

and the condition (7) according to which the quantity $\zeta = \mu + eEz$ should not depend on any coordinates. Taking into account that in the one-miniband approximation

$$n_e^{(0)}(\zeta_0) = m_\perp \zeta_0/\pi d\hbar^2 \tag{17}$$

and substituting (12) in (16) (thus neglecting the oscillations of ζ under the variation of the fields), we find the dependence of ζ on electric and magnetic fields:

$$\zeta = (\hbar\omega_E/2) + (2\hbar\omega_E\zeta_0)^{1/2}[1 + (\hbar\omega_E/24\zeta_0) + (\omega_H/\omega_E)(\hbar\omega_H/24\zeta_0)]^{1/2}. \tag{18}$$

To avoid ambiguity, note that the above consideration is based on assumption (2) and therefore the passage to the limit $E \to 0$ in formula (18) is inadmissible.

Assuming further that the conditions

$$\hbar\omega_E < \zeta_0 \qquad \hbar\omega_E/24\zeta_0 \ll 1 \qquad (\omega_H/\omega_E)(\hbar\omega_H/24\zeta_0) \ll 1 \tag{19}$$

are satisfied and using (12), (13) and (18), we finally get

$$\Omega^{\rm mon} = \frac{m_\perp V_0}{6\pi\hbar^2 d}\left(\frac{2\zeta_0}{\hbar\omega_E}\right)^{1/2}\left[\left(\frac{\hbar\omega_H}{2}\right)^2 + \left(1 - \frac{8\zeta_0}{\hbar\omega_E}\right)\left(\frac{\hbar\omega_E}{2}\right)^2 - (\pi T)^2\right] \tag{20}$$

$$\Omega^{\rm osc} = \frac{m_\perp V_0 \omega_H^3}{2\pi^4 \omega_E d}\sum_l \left\{ l^{-3}\Psi_1\left(\frac{2\pi^2 l T}{\hbar\omega_H}\right)\left[\pi l \frac{\omega_E}{\omega_H}\sqrt{\frac{2\zeta_0}{\hbar\omega_E}}\cos\left(2\pi l \frac{\omega_E}{\omega_H}\sqrt{\frac{2\zeta_0}{\hbar\omega_E}}\right)\right.\right.$$
$$\left.\left. - \Psi_2\left(\frac{2\pi^2 l T}{\hbar\omega_H}\right)\sin\left(2\pi l\frac{\omega_E}{\omega_H}\sqrt{\frac{2\zeta_0}{\hbar\omega_E}}\right)\right]\right\}. \tag{21}$$

4. Magnetic susceptibility

Using the expressions (20) and (21) it is easy to find the different thermodynamic characteristics of the electron gas in the SL, such as entropy, electric moment and magnetization. It is the latter quantity that is the object of our interest here—more precisely, the magnetic susceptibility χ, which is defined as the derivative of the magnetization M with respect to the magnetic field

$$\chi = (\partial M/\partial H)_{T,\zeta_0,E} = -V_0^{-1}(\partial^2\Omega/\partial H^2)_{T,\zeta_0,E}. \tag{22}$$

Note that in evaluating the oscillating part of χ we must differentiate, as usual, only the most rapidly varying functions in (21), namely, the sine and cosine in the terms of the sum. As a result we get the following expressions for the monotonic ($\bar{\chi}$) and oscillating ($\tilde{\chi}$) parts of the magnetic susceptibility:

$$\bar{\chi} = -\frac{m_\perp \mu_B^2}{3\pi\hbar^2 d}\left(\frac{2\zeta_0}{\hbar\omega_E}\right)^{1/2} \tag{23}$$

$$\tilde{\chi} = \bar{\chi}\frac{48\zeta_0}{\hbar\omega_H}\frac{\omega_E}{\omega_H}\sum_l \left\{\Psi_1\left(\frac{2\pi^2 lT}{\hbar\omega_H}\right)\left[\cos\left(2\pi l\frac{\omega_E}{\omega_H}\sqrt{\frac{2\zeta_0}{\hbar\omega_E}}\right)\right.\right.$$
$$\left.\left. + \frac{1}{\pi l}\left(\frac{\omega_H}{2\omega_E}\right)^{1/2}\left(\frac{\hbar\omega_H}{\zeta_0}\right)^{1/2}\Psi_2\left(\frac{2\pi^2 lT}{\hbar\omega_H}\right)\sin\left(2\pi l\frac{\omega_E}{\omega_H}\sqrt{\frac{2\zeta_0}{\hbar\omega_E}}\right)\right]\right\} \tag{24}$$

where $\mu_B = e\hbar/2m_\perp c$ is the effective Bohr magneton.

From (23) it follows that the monotonic part of the diamagnetic susceptibility does not depend on temperature, but depends essentially on the electric field strength, decreasing with increase in the field by the law $\bar{\chi} \propto E^{-1/2}$. It should also be noted that according to (23) (and bearing in mind (17)), the quantity $\bar{\chi}$ increases with electron density $n_e^{(0)}$ proportional to $\sqrt{n_e^{(0)}}$, whereas the Landau diamagnetic susceptibility of the three-dimensional electron gas $\bar{\chi}_L \propto n_e^{1/3}$ [25]. This difference arises from the effective reduction of the dimensionality of the electron gas (its quasi-zero-dimensionality) caused by the quantization of the electron motion in external fields.

The Wannier–Stark quantization also affects essentially the Pauli paramagnetic magnetization, which is the result of the spin magnetic moment of the electrons. Let us calculate the corresponding magnetic susceptibility χ_p assuming $\hbar\omega_H \ll T$. In this case we can neglect the effect of the magnetic field quantization of the electron orbits.

With regard to the electron spin s the energy spectrum of electrons in the SL under a quantized electric field may be written as

$$\varepsilon_{mp_\perp s} = m\hbar\omega_E + p_\perp^2/2m_\perp - \mu_B s \cdot H \tag{25}$$

where $p_\perp$ is the electron momentum in the xy plane and $s = 1(\uparrow), -1(\downarrow)$ is the spin quantum number. At $H = 0$ the density of states of the electrons with a fixed value of s is given by

$$D(\varepsilon) = \sum_m \sum_{p_\perp} \delta(\varepsilon - \varepsilon_{mp_\perp}) = (m_\perp V_0/2\pi\hbar^2 d)[\varepsilon/\hbar\omega_E] \tag{26}$$

where $[x]$ denotes the integral part of the number x. Then, for the paramagnetic magnetization of the electron gas for $\zeta \gg \mu_B H$ we obtain

$$M_p = M_\uparrow - M_\downarrow = (\mu_B/V_0) \int D(\varepsilon)[f_0(\varepsilon - \mu_B H) - f_0(\varepsilon + \mu_B H)]\,d\varepsilon$$
$$= (m_\perp \mu_B^2 H/\pi\hbar^2 d)[\zeta/\hbar\omega_E] \tag{27}$$

where ζ is the chemical potential at $H = 0$. The dependence of ζ on the electric field E is given by (18), and therefore, at $\zeta_0 \gg \hbar\omega_E$, we get the following expression for the paramagnetic susceptibility:

$$\chi_p = \left(\frac{\partial M_p}{\partial H}\right)_{T,\zeta_0,E} = \frac{m_\perp \mu_B^2}{\pi\hbar^2 d}\left[\sqrt{\frac{2\zeta_0}{\hbar\omega_E}}\right] \tag{28}$$

where the quantity ζ_0 is determined by (17).

From (28) it follows that on changing the electric field strength the quantity χ_p experiences step-like jumps corresponding to the alteration of the magnitude $[(2\zeta_0/\hbar\omega_E)^{1/2}]$ by unity. It occurs every time the value of $(2\zeta_0/eEd)^{1/2}$ becomes an integer. As follows from (26), such a dependence of $\chi_p(E)$ is the consequence of the step-like behaviour of the electron density of states as a function of energy in the SL in the presence of a quantizing electric field.

However, the most interesting effects caused by the Wannier–Stark quantization can be observed in the oscillating part of χ. As seen from (24), the quantity $\tilde{\chi}$ can oscillate with a change not only of the magnetic field strength, but of the electric field strength as well, the oscillatory variation of $\tilde{\chi}$ as a function of H being characterized by the period

$$\Delta(1/H) = \mu_B(2/eEd\zeta_0)^{1/2} \tag{29}$$

depending on the electric field strength and differing from the period of the de Haas–van Alphen oscillations. Still more specific are the oscillations of $\tilde{\chi}$ with a change of the electric field strength. As follows from (24) they arrange evenly on the scale $\sqrt{E}$ with a period proportional to the magnitude H:

$$\Delta(E^{1/2}) = 2\mu_B H/(2ed\zeta_0)^{1/2}. \tag{30}$$

From the physical standpoint the rise of the above-mentioned oscillations is quite easy to interpret if we turn to figure 1 and consider how the filling of the electron states is being changed with the variation of the electric and magnetic fields. One should take into account the fact that in the presence of Wannier–Stark quantization the level of the chemical potential ζ in the SL, as seen from (18), itself depends on the electric and magnetic field strengths. If condition (19) is satisfied, the magnetic field variation does not change essentially the quantity ζ, and assuming $\zeta_0 \gg \hbar\omega_E$ we have $\zeta = \sqrt{2\zeta_0\hbar\omega_E}$. Therefore, when the electric field E is steady, the number of filled Landau levels is changed by unity with a magnetic field variation (more exactly, with the reverse field variation) $\Delta(1/H)$ by the amount $2\mu_B/\zeta$, and thus we get to (29).

On the contrary, if the magnetic field is kept steady and the electric field is varied, with the increase of E the level of ζ will also increase, but more slowly than the distance between the Wannier–Stark levels, and one of the Landau levels will cross the ζ level. Designating by E_1 and E_2 the two values of the electric field for which the number of Landau levels (with energies less than or equal to ζ) is equal to n and $n+1$, respectively, we will have

$$\frac{\sqrt{2\zeta_0 e E_1 d}}{\hbar\omega_H} = n \qquad \frac{\sqrt{2\zeta_0 e E_2 d}}{\hbar\omega_H} = n+1.$$

Subtracting the first equality from the second one we get the formula (30) for the oscillation period $\Delta(E^{1/2}) = E_2^{1/2} - E_1^{1/2}$.

The temperature dependence of the amplitudes of the oscillations is determined, as in the usual de Haas–van Alphen effect, by the function $\Psi_1(2\pi^2 lT/\hbar\omega_H)$ because of the relative smallness of the second term in brackets in (24) (as seen from (14), (15) and from the latter condition in (19)) in the region of strong magnetic fields ($2\pi^2 T \ll \hbar\omega_H$). In this region the exponential decrease of terms in the series (24) begins only from $l > l_0 \sim \hbar\omega_H/T$ and the amplitude of the oscillations is determined by the sum of a large number of cosine terms in (24) for which $l \sim l_0 \gg 1$. The number of such terms is of the order of magnitude of the same l_0. As a result we get the following estimation:

$$\tilde{\chi}^{\max}/\bar{\chi} \sim 48(\hbar\omega_E/T)(\zeta_0/\hbar\omega_H). \tag{31}$$

Hence, provided the conditions (2) and (19) are satisfied, the amplitude of the oscillating part of the magnetic susceptibility χ is large compared with the monotonic one.

However, one should bear in mind that electron scattering on different structural defects (such as impurities, interface roughnesses and well width fluctuations) can bring about a decrease of the amplitudes of the oscillations. A rigorous treatment of the effects of these processes on the behaviour of the oscillating part of χ is a separate problem and is outside the scope of the present paper. But a qualitative estimation of the influence of scattering processes on $\tilde{\chi}$ is easy to perform if we suppose, following Dingle [41], that they do not change the systematics of the discrete energy levels, but merely lead to their broadening by a magnitude of the order of $\hbar/\tau$. In this case the collisions of electrons with scatterers play the same role as temperature does, and the scattering can be taken into account by substituting the effective temperature $T_{\text{eff}} = T + \hbar/\tau$ for T in the function $\Psi_1(2\pi lT/\hbar\omega_H)$ in (24). As a result, every lth harmonic of the magnetic susceptibility acquires the multiplier $\exp(-2\pi l/\omega_H\tau)$, known as the Dingle factor and describing the decrease of the oscillation amplitudes of χ. With allowance for this factor, the exponential decrease of terms in the series (24) will begin from $l = l_0 = \min(\hbar\omega_H/2\pi^2 T, \omega_H\tau/2\pi)$. At low temperature ($T = 4.2$ K) for parameters characteristic of the GaAs–AlGaAs SL, $\hbar/\tau \lesssim \pi T$, and the decrease of the amplitude of the oscillations at the expense of scattering processes will not be very essential, so that the estimation (31) will remain valid at least as an order of magnitude.

The graphs in figures 2 and 3 give a visual presentation of the amplitude and shape of the oscillation peaks of χ and also illustrate clearly the dependence of the oscillation periods on the strength of the fields E and H; the graphs have been computed by using typical values of the parameters of a doped GaAs–AlGaAs SL: $m_\perp = 0.01 m_0$ (m_0 is the free-electron mass), $d = 100$ Å, $\Delta = 50$ meV and $n_e = 3 \times 10^{17}$ cm^{-3}.

5. Conclusions

In this paper we have shown that the equilibrium magnetic properties of the electron gas in a semiconductor SL change essentially by applying quantizing electric and magnetic fields

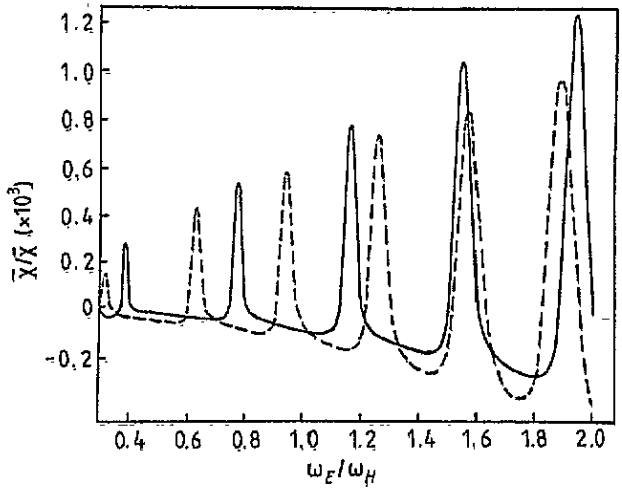

Figure 2. The periodic dependence of $\tilde{\chi}/\bar{\chi}$ on ω_E/ω_H at $T = 4.2$ K and two different constant values of the electric field E: $E = 15$ kV cm^{-1} (full curve) and $E = 10$ kV cm^{-1} (broken curve).

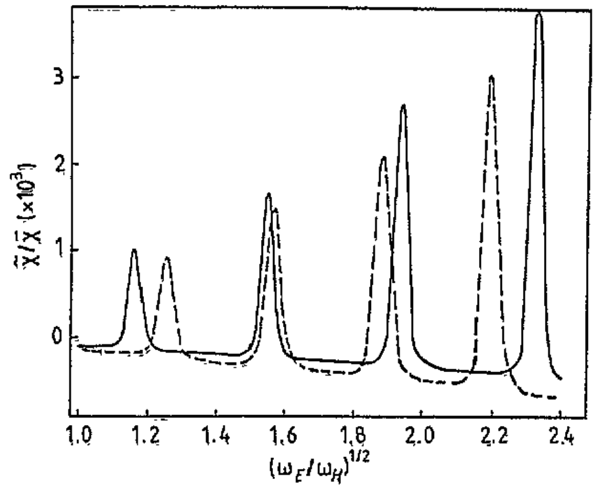

Figure 3. The periodic dependence of $\tilde{\chi}/\bar{\chi}$ on $(\omega_E/\omega_H)^{1/2}$ at $T = 4.2$ K and two different constant values of the magnetic field H: $H = 15$ kOe (full curve) and $H = 10$ kOe (broken curve).

along the SL axis. The most important changes are the dependence of the monotonic part of diamagnetic susceptibility of the electron gas on the electric field strength and the step-like dependence of the paramagnetic susceptibility on E in the region of weak magnetic fields. An experimental investigation of these dependences would allow one to single out unambiguously the contribution of free carriers to the magnetic susceptibility of the SL, which, generally, also includes the susceptibilities of the host crystal atoms and of different structural defects. The results of the given measurements could be used for determining some characteristic parameters of the SL, such as the concentration of electrons and their effective mass. It should be noted that such measurements have the advantage over purely electric ones in that the results obtained do not depend on the carrier scattering mechanism.

The Wannier–Stark quantization also affects the oscillating part of the magnetic susceptibility. The most essential effects (which are at the same time available for observation) are the rise of a new oscillation period of magnetic susceptibility as a function of the magnetic field different from that of the de Haas–van Alphen oscillations and the oscillatory behaviour of χ as a function of the electric field having a periodicity in $\sqrt{E}$. A distinctive feature of both types of oscillations is the possibility of regulating their periods by changing the magnitude of the constant external fields applied to the SL: the electric field in the first case and the magnetic field in the second.

We believe that the considered effects give new possibilities to investigate the electron energy spectrum in a semiconductor SL, suggesting a fundamental method for the experimental study of the Wannier–Stark quantization phenomenon.

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